Nonlinear Feedback Control for Remote Orbital Capture

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The problem of detumbling a freely spinning and precessing axisymmetric target satellite with feedback control by an axisymmetric retriever spacecraft or orbital maneuvering vehicle is considered. Equations of motion for the two-body system are derived using Euler's dynamical equations in conjunction with the methodology developed by Hooker and Margulies. A Liapunov technique is employed to derive a nonlinear feedback control law for the thruster torques applied by the orbital maneuvering vehicle and the motor torques applied at the joint to drive the two-body system to a final spin-stabilized state asymptotically. State and control histories are presented to illustrate that the detumbling process is quite benign and that the required control magnitudes are small. The loads at the joint are determined and found to be within acceptable bounds during detumbling.

Nomenclature

$\hat{e}_1, \hat{e}_2, \hat{e}_3$ = unit vector set arbitrarily placed at the mass center of body 0 (OMV) and aligned with its principal axes
F'_λ = nongravitational external force on body λ
F^H_λ = interaction force on body λ transmitted through joint j
$\hat{g}_i$ = unit vectors representing the axes at the joints of an n-body system about which rotation is possible
$\hat{i}, \hat{j}, \hat{k}$ = unit vector set representing inertial planetocentric coordinate system
J_λ = set of joints on body λ
$\mathscr{L}_{\lambda\mu}$ = vector from the mass center of a body λ to the joint on body λ leading to a body μ
m = total mass of the n-body system
m_λ = mass of a component body λ
n = number of bodies in the multibody system
$\hat{n}_1, \hat{n}_2, \hat{n}_3$ = unit vector set arbitrarily placed at the mass center of body 1 (target) and aligned with its principal axes
r = number of degrees of rotational freedom in the n-body system
r_λ = vector from the system mass center to the mass center of body λ
T'_λ = nongravitational external torque on body λ
$T^C_{\lambda j}$ = constraint torque on body λ at joint j
$T^{SD}_{\lambda j}$ = spring damper torque on body λ at joint j
t = time
u = five-element control vector
x = five-element state vector
${}_1x_6$ = x augmented with a sixth element
y = three-element joint translational state vector
1 = unit dyadic
γ = planet's gravitational constant
γ_i = angle of rotation about axis $\hat{g}_i$
ρ = planetocentric position vector of n-body system mass center
$\hat{\rho}$ = unit vector in the direction of ρ
ρ_λ = planetocentric position vector of the mass center of body λ
ϕ_λ = inertia dyadic of body λ about mass center
ψ = precession angle of body 1 (target) about $\hat{e}_3$
ω_λ = inertial angular velocity of body λ

Subscripts

i = degrees of rotational freedom relative to body 0 ($i = 1, 2, \ldots, r-3$)
j = an element of J_λ
λ = an arbitrary body in an n-body system
μ = an arbitrary body in an n-body system

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Introduction

THE servicing and repair of satellites in orbits beyond the direct reach of the Space Shuttle requires a teleoperator spacecraft or orbital maneuvering vehicle (OMV) to dock with these target satellites and despin or detumble them if they are spin stabilized or have experienced a control system failure. See Fig. 1. Docking followed by despinning or detumbling is defined here as capture. Docking is accomplished by driving a grappling device on the OMV to a state of rest relative to a docking point on the target so that the two bodies can be connected. Despinning or detumbling is accomplished by applying torques to the target through the grappling device, while simultaneously applying torques to the OMV to control the absolute motion of the two-body system.

Previous work[1,2] has considered the detumbling aspect of capture, but without treating the requirement to control the absolute motion of the two-body system. To investigate this requirement with realistic mass properties of the target satellite and the OMV, Conway et al.[3] proposed the problem of detumbling a freely spinning and precessing axisymmetric target satellite with an axisymmetric OMV (Fig. 1). Dynamic coupling was clearly demonstrated, but absolute motion was not controlled and detumbling was not accomplished.

In the present work, a nonlinear feedback control law is derived by Liapunov methods that solves the detumbling problem proposed by Conway et al.[3] with the additional condition that the joint connecting the target to the OMV is capable of translating along the surface of the OMV. There are at least two reasons for allowing joint translation: 1) the state of the target may not be accurately known prior to rendezvous, making it impossible to preposition the joint; and

2) by positioning the joint during detumbling such that the symmetry axis of the OMV becomes aligned with the symmetry axis of the target, the final configuration can be spin stabilized.

This is certainly not the first application of Liapunov theory to spacecraft stabilization and control. Early results were presented by Likins[4] Mortensen,[5] and Flanagan and Rangarajan.[6] Other authors, including Bainum and Sellappan[7] and Kaplan,[8] even consider special detumbling problems. Here, the problem of detumbling a rigid body through the action of a second rigid body connected to it is treated with a Liapunov analysis to extend the body of literature on spacecraft detumbling. To formulate the control problem, the two-body system configuration and the initial and desired final states must be defined and the equations of motion derived.

Problem Formulation

In Fig. 1, the OMV is designated as body 0 and the target as body 1. The $\hat{i}, \hat{j}, \hat{k}$ unit vectors represent an inertial planetocentric coordinate system. A principal axis coordinate system of the OMV emanating from its mass center is represented by the $\hat{e}_1, \hat{e}_2, \hat{e}_3$ unit vectors. The $\hat{n}_1, \hat{n}_2, \hat{n}_3$ unit vectors are used similarly in the target. Both principal axis coordinate systems are considered fixed in their respective bodies. The connecting joint may be located with respect to the mass center of body 0 by the vector $[0, \ell, c]$ using the $\hat{e}$ unit vectors. ℓ is the variable joint position in the $\hat{e}_2$ direction as shown in Fig. 1 and c is the constant distance of the joint from the OMV mass center in the $\hat{e}_3$ direction.

Since the target is precessing about its own angular momentum vector at a rate $\dot{\psi}$, the OMV is positioned relative to the target so that the target's mass center is on the OMV principal axis represented by $\hat{e}_3$ and the target's angular momentum vector is parallel to $\hat{e}_3$. The OMV is then spun about the $\hat{e}_3$ axis at the same rate $\dot{\psi}$. The target's cone angle γ_1 and the distance in the $\hat{n}_3$ direction from the target's mass center to the surface of the OMV determine the position of the connecting joint for docking. The target's cone angle γ_1, precession rate $\dot{\psi}$, spin rate $\dot{\gamma}_2$, and mass properties are interrelated as shown by Greenwood[9] for steady spin and precession.

The two rotational degrees of freedom required at the joint are now clear. The first permits a rotation γ_1 about an axis $\hat{g}_1$ parallel to the vector $\hat{e}_1$. The second permits a rotation γ_2 about an axis $\hat{g}_2$ in the $\hat{n}_3$ direction. Furthermore, $\hat{g}_1$ and $\hat{g}_2$ are mutually orthogonal. With the two bodies docked in this manner, a dynamically stable configuration results. With no torques or forces acting on the system, the OMV remains in pure spin and the target remains in free spin and precession. From this initial docked configuration, the problem is to drive the system with a set of feedback controls to a final spin-stabilized state with the connecting joint located by a vector $[0, 0, c]$ using the $\hat{e}$ unit vectors, and the angle γ_1 and the angular rate $\dot{\gamma}_2$ reduced to steady-state values of zero.

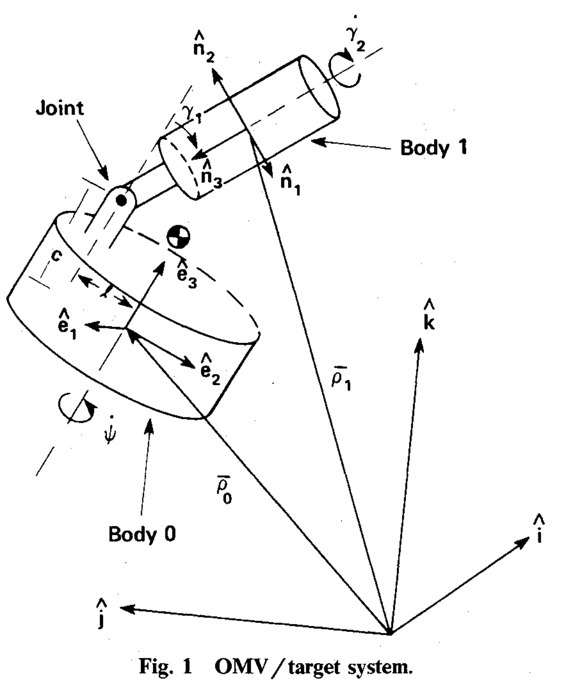

Fig. 1 OMV/target system.

Equations of Motion

The attitude equations of motion of the OMV/target system can be derived from the Eulerian-based equations of motion for multibody systems given by Hooker and Margulies[10] and Hooker[11] as extended to account for joint translation by Conway and Widhalm.[12] These equations were presented by Conway and Widhalm[13] and Widhalm[14] and are repeated here for convenience as follows:

$$\begin{bmatrix} a_{00} \boldsymbol{a}_{01} \boldsymbol{a}_{02} \\ \\ \boldsymbol{a}_{10} a_{11} a_{12} \\ \boldsymbol{a}_{20} a_{21} a_{22} \end{bmatrix} \begin{bmatrix} \dot{\omega}_0 \\ \\ -\ddot{\gamma}_1 \\ \ddot{\gamma}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_0^* + \boldsymbol{E}_1^* - m_1 \mathscr{L}_{01} \times \left(\ddot{\boldsymbol{D}}_{01}^R + 2\omega_0 \times \dot{\boldsymbol{D}}_{01}^R \right) \\ + \boldsymbol{D}_{10} \times m \left(\ddot{\boldsymbol{D}}_{01}^R + 2\omega_0 \times \dot{\boldsymbol{D}}_{01}^R \right) \\ \hat{g}_1 \cdot \left[\boldsymbol{E}_1^* + \boldsymbol{D}_{10} \times m \left(\ddot{\boldsymbol{D}}_{01}^R + 2\omega_0 \times \dot{\boldsymbol{D}}_{01}^R \right) \right] \\ \hat{g}_2 \cdot \left[\boldsymbol{E}_1^* + \boldsymbol{D}_{10} \times m \left(\ddot{\boldsymbol{D}}_{01}^R + 2\omega_0 \times \dot{\boldsymbol{D}}_{01}^R \right) \right] \end{bmatrix} \tag{1}$$

Equation (1) for the two-body system considered here follows from the equations for n bodies with r rotational degrees of freedom where

$$\begin{aligned} a_{00} &= \sum_{\lambda} \sum_{\mu} \Phi_{\lambda\mu}, \quad \text{a dyadic} \\ \boldsymbol{a}_{0k} &= \sum_{\lambda} \sum_{\mu} \epsilon_{k\mu} \Phi_{\lambda\mu} \cdot \hat{g}_k \quad \text{a vector} \\ \boldsymbol{a}_{i0} &= \hat{g}_i \cdot \sum_{\lambda} \sum_{\mu} \epsilon_{i\lambda} \Phi_{\lambda\mu} \quad \text{a vector} \\ a_{ik} &= \hat{g}_i \cdot \sum_{\lambda} \sum_{\mu} \epsilon_{i\lambda} \epsilon_{k\mu} \Phi_{\lambda\mu} \cdot \hat{g}_k \quad \text{a scalar} \end{aligned} \tag{2}$$

$\epsilon_{i\mu} = 1$, if $\hat{g}_i$ belongs to a joint anywhere on the chain of bodies connecting body μ and the reference body

$= 0$, otherwise (e.g., $\mu = 0$)

$$\Phi_{\lambda\lambda} = \Phi_\lambda + m_\lambda \left[D_\lambda^2 1 - D_\lambda D_\lambda \right] + \sum_{\mu \neq \lambda} m_\mu \left[D_{\lambda\mu}^2 1 - D_{\lambda\mu} D_{\lambda\mu} \right] \tag{3}$$

$$\Phi_{\lambda\mu}(\mu \neq \lambda) = -m \left[D_{\mu\lambda} \cdot D_{\lambda\mu} 1 - D_{\mu\lambda} D_{\lambda\mu} \right] \tag{4}$$

$$D_\lambda = -\sum_{\mu \neq \lambda} m_\mu m^{-1} \mathscr{L}_{\lambda\mu} \tag{5}$$

$$D_{\lambda\mu} = D_\lambda + \mathscr{L}_{\lambda\mu} \tag{6}$$

$\mathscr{L}_{\lambda\mu}$ is the vector from the center of mass of body λ to the joint on body λ leading to body μ. $\boldsymbol{E}_\lambda^*$ is determined by

$$\boldsymbol{E}_\lambda^* = \boldsymbol{E}_\lambda - \sum_{\mu} \Phi_{\lambda\mu} \cdot \sum_{k} \epsilon_{k\mu} \dot{\gamma}_k \dot{\hat{g}}_k \tag{7}$$

and E_λ is the vector

$$E_\lambda = 3\gamma\rho^{-3}\hat{\rho} \times \Phi_{\lambda\lambda} \cdot \hat{\rho} - \omega_\lambda \times \Phi_{\lambda\lambda} \cdot \omega_\lambda + T'_\lambda + \sum_{j\in J_\lambda} T^{SD}_{\lambda j} + D_\lambda \times F'_\lambda + \sum_{\mu\neq\lambda} D_{\lambda\mu} \times \left\{ F'_\mu + m\omega_\mu \times \left[\omega_\mu \times D_{\mu\lambda}\right] + m\gamma\rho^{-3}[1 - 3\hat{\rho}\hat{\rho}] \cdot D_{\mu\lambda} \right\} \tag{8}$$

In Eq. (1), $\omega_0 = \omega_{01}\hat{e}_1 + \omega_{02}\hat{e}_2 + \omega_{03}\hat{e}_3$, and γ_1 and γ_2 are the angles of rotation previously defined. All vectors are expressed in the $\hat{e}$ system and superscript R implies that the indicated time differentiation is performed relative to the rotating principal axis frame in which the $\hat{e}$ unit vectors are fixed. References 11 and 12 contain a detailed derivation of these equations and explanation of the notation employed.

To use Eq. (1), the relative acceleration $\ddot{D}^R_{01}$ or relative velocity $\dot{D}^R_{01}$ of the joint with respect to the mass center of body 0 must be prescribed. Control torques and forces, which appear explicitly in E^*_0 and E^*_1 must be specified. To determine the inertial orientation of the two-body system, a set of first order equations, such as the four Euler parameter equations, as well as two equations for determining γ_1 and γ_2, must be added to the system of Eq. (1).

In deriving these equations from Refs. 10-12, the constraint torque and force at the joint have been eliminated. These quantities can be determined, however, as originally described in Refs. 10 and 11. Explicit expressions for the constraint force and torque of the OMV target system of Fig. 1 were also given by Conway and Widhalm[13] and are repeated here,

$$F^H_{01} = m_0\ddot{r}_0 \tag{9}$$

$$T^C_{11} = \phi_{10} \cdot \dot{\omega}_0 + \phi_{11} \cdot \dot{\omega}_1 - E_1 - D_{10} \times m\left(\ddot{D}^R_{01} + 2\omega_0 \times \dot{D}^R_{01}\right) \tag{10}$$

where r_0 is the vector from the system mass center to the mass center of body 0. r_0 may be rewritten as

$$r_0 = \frac{m_1}{m}(\mathscr{L}_{10} - \mathscr{L}_{01}) = \frac{m_1}{m}\mathscr{L} \tag{11}$$

and differentiated twice with respect to time for use in Eq. (9). From Eq. (9), the component of the constraint force in the $\hat{e}_2$ direction is the force required to translate the joint as specified. Gravitational effects have been ignored in writing these two equations and are ignored in the subsequent analysis.

Liapunov Feedback Control

The detumbling problem described above can be solved by a feedback control approach in which three external (thruster) torques and two internal (motor) torques, forming a five-element control vector u are nonlinear functions of the system state variables. In this section, the axes are defined about which the five controls are applied, so that the equations of motion take a simpler form more suitable to a Liapunov analysis. Liapunov's direct method is applied to derive the desired nonlinear feedback control law that, when applied to the dynamic system, reduces the equations of motion to a linear system globally asymptotically stable with respect to the spin-stabilized equilibrium.

To specify the five-element control vector u, Eq. (1) is first written in a more compact form as

$$A\dot{x} = F^* \tag{12}$$

The 5×5 matrix on the left-hand side of Eq. (1) is defined as the matrix A. The five-element vector on the right-hand side of Eq. (1) is defined as F^* and

$$\dot{x} = [\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4\ \dot{x}_5]^T = [\dot{\omega}_{01}\ \dot{\omega}_{02}\ \dot{\omega}_{03}\ -\ddot{\gamma}_1\ \ddot{\gamma}_2]^T \tag{13}$$

Now, because the two gimbal axes at the joint are orthogonal and because the internal torques applied at the joint cancel from the first three elements of F^*, the control vector u can be selected so that Eq. (12) can be written as

$$A\dot{x} = F + u \tag{14}$$

Components u_1, u_2, and u_3 of u are external (thruster) torques applied about the principal axes of the OMV as represented by the $\hat{e}$ unit vectors. Components u_4 and u_5 of u are internal (motor) torques applied about the gimbal axes $\hat{g}_1$ and $\hat{g}_2$ having directions $\hat{e}_1$ and $\hat{n}_3$, respectively. The control vector u is assumed to be continuous and the system mass properties are assumed to be unchanged by thruster firings. Finally, we can write

$$\dot{x} = A^{-1}F + A^{-1}u \tag{15}$$

since the existence of A^{-1} is guaranteed in this case.

Also required is the kinematical equation

$$\dot{x}_6 = x_4 \tag{16}$$

where x_6 is $-\gamma_1$. Consequently, we define the augmented state vector, ${}_1x_6$, to contain the vector x plus the sixth element x_6.

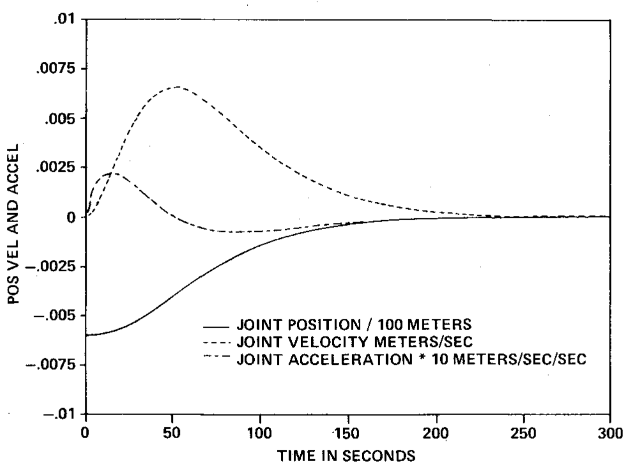

Fig. 2 Joint motion histories.

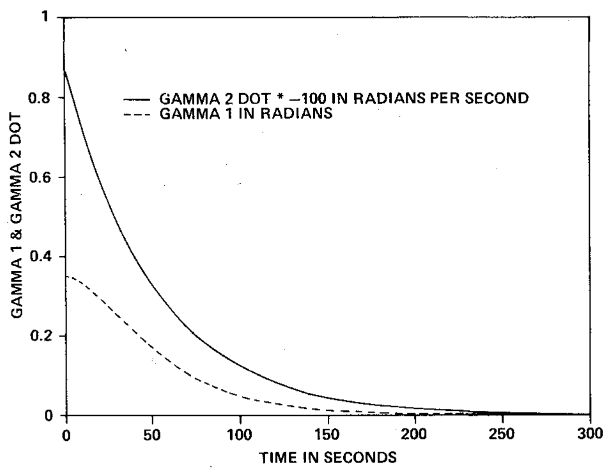

Fig. 3 Body 1 γ_1 and $\dot{\gamma}_2$ histories.

To derive the control vector $\boldsymbol{u}$ as a function of the vector ${}_1\boldsymbol{x}_6$ and variables characterizing the joint motion, a lemma presented by Vidyasagar[15] applicable to autonomous or periodic systems is required. Equation (15) is nonautonomous because translational joint motion appears as a specified function of time. However, by specifying the joint motion as a third-order linear system, asymptotically stable with respect to the desired final joint position $[0, 0, c]$, Eqs. (15) and (16) can be augmented with this third-order system to form an autonomous system. Carrying this out, let

$$\mathscr{L}_{01} \cdot \hat{e}_2 = y_1, \quad \dot{\mathscr{L}}_{01}^R \cdot \hat{e}_2 = y_2$$
$$\ddot{\mathscr{L}}_{01}^R \cdot \hat{e}_2 = y_3 \tag{17}$$

Then, an autonomous system is formed by writing

$$\dot{\boldsymbol{x}} = A^{-1}\boldsymbol{F} + A^{-1}\boldsymbol{u}$$
$$\dot{x}_6 = x_4, \quad \dot{\boldsymbol{y}} = D\boldsymbol{y} \tag{18}$$

where D is a negative definite matrix. The joint motion contained in the matrix A and the vector $\boldsymbol{F}$ is now specified by the vector $\boldsymbol{y}$, removing time explicitly from the system.

Considering $V({}_1\boldsymbol{x}_6, \boldsymbol{y})$ to be a candidate Liapunov function, lemma 81 developed by Vidyasagar[15] in his discussion of Liapunov stability theory is now stated for the system of Eq. (18), as follows:

"Let $V({}_1\boldsymbol{x}_6, \boldsymbol{y})$ be continuously differentiable and suppose that for some $d \geq 0$, the set

$$\Omega_d = \{{}_1\boldsymbol{x}_6, \boldsymbol{y}\text{: } V({}_1\boldsymbol{x}_6, \boldsymbol{y}) \leq d\}$$

is bounded. Suppose that V is bounded below over the set Ω_d and that $\dot{V}({}_1\boldsymbol{x}_6, \boldsymbol{y}) \leq 0$ for all ${}_1\boldsymbol{x}_6$ and $\boldsymbol{y}$ in Ω_d. Let S denote the subset of Ω_d defined by

$$S = \{{}_1\boldsymbol{x}_6, \boldsymbol{y} \in \Omega_d\text{: } \dot{V}({}_1\boldsymbol{x}_6, \boldsymbol{y}) = 0\}$$

and let M be the largest invariant set of the system [Eq. (18)] contained in S. Then, whenever

$${}_1\boldsymbol{x}_6(0), \boldsymbol{y}(0) \in \Omega_d$$

the solution of the system [Eq. (18)] approaches M as $t \to \infty$."

To clarify, since the system of Eq. (18) is autonomous, any state trajectory of the system is an invariant set. Also, in contrast to those Liapunov stability theorems that require V to be positive definite, this is not the case here, but the candidate Liapunov function to be considered next is positive definite.

To derive a nonlinear feedback control law that drives the two-body system to a spin-stabilized equilibrium, consider the candidate Liapunov function

$$V = \left(\tfrac{1}{2}\right)\boldsymbol{x}^T I \boldsymbol{x} + \left(\tfrac{1}{2}\right) K x_6^2 + \boldsymbol{y}^T R \boldsymbol{y} \tag{19}$$

where I is the identity matrix, K a positive constant, and R a positive definite constant matrix. Differentiating with respect to time gives

$$\dot{V} = \boldsymbol{x}^T I \dot{\boldsymbol{x}} + K x_6 \dot{x}_6 + \dot{\boldsymbol{y}}^T R \boldsymbol{y} + \boldsymbol{y}^T R \dot{\boldsymbol{y}} \tag{20}$$

Substituting from Eqs. (18) yields

$$\dot{V} = \boldsymbol{x}^T I [A^{-1}\boldsymbol{F} + A^{-1}\boldsymbol{u}] = + K x_6 x_4 + \boldsymbol{y}^T [D^T R + R D]\, \boldsymbol{y} \tag{21}$$

However, since D is specified to be negative definite, the familiar Liapunov equation can be written,

$$D^T R + R D = -Q \tag{22}$$

where Q is a positive definite matrix. Therefore, $\boldsymbol{y}^T[D^T R + R D]\boldsymbol{y}$ is negative definite. To make $\dot{V}$ at least negative semidefinite, choose

$$\boldsymbol{u} = -\boldsymbol{F} + A[0\ 0\ 0\ -Kx_6\ 0]^T - AB\boldsymbol{x} \tag{23}$$

If the matrix B is positive definite, then $\dot{V}$ is negative semidefinite in x_6. However, if B is diagonal with positive elements (except $B_{33} = 0$), $\dot{V}$ is negative semidefinite in x_3 and x_6. Then, from the above lemma, the system [Eq. (18)], with the control $\boldsymbol{u}$ of Eq. (23), is asymptotically stable with respect to the largest invariant set contained in the x_3, x_6 plane. However, any nonzero x_6 results in a nonzero control $\boldsymbol{u}$, which causes a departure from the x_3, x_6 plane. Therefore, the largest invariant set contained in the x_3, x_6 plane is the x_3 axis, which represents the spin-stabilized equilibrium. We conclude then that Eq. (23) is the desired nonlinear feedback control law for the system of Eq. (18).

Substituting Eq. (23) for $\boldsymbol{u}$ in the system of Eq. (18) results in a linear system written as

$$\dot{\boldsymbol{x}} = [0\ 0\ 0\ -Kx_6\ 0]^T - B\boldsymbol{x}$$
$$\dot{x}_6 = x_4 \tag{24}$$
$$\dot{\boldsymbol{y}} = D\boldsymbol{y}$$

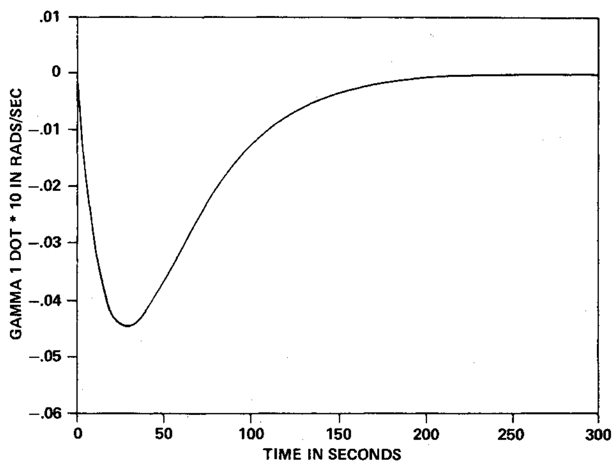

Fig. 4 Body 1 $\dot{\gamma}_1$ history.

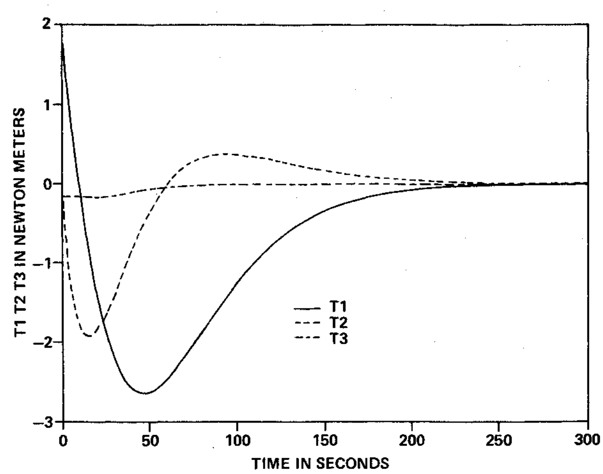

Fig. 5 External torque histories.

Table 1 Spacecraft mass properties

Spacecraft	Mass, kg	I_1, kg · m²	I_2, kg · m²	I_3, kg · m²
Target	1000	1000	1000	1100
Retriever	4500	6400	6400	11,800

Table 2 Initial conditions

$\omega_{01} = 0.0$	$\dot{\gamma}_1 = 0.0$
$\omega_{02} = 0.0$	$\dot{\gamma}_2 = -0.009$ rad/s
$\omega_{03} = -0.102$ rad/s	$\gamma_1 = 0.349$ rad
Initial joint position $= \mathscr{L}_{01} = -0.599\hat{e}_2 + 0.62\hat{e}_3$ m	
Length (const) $\mathscr{L}_{10} = (1.75\text{ m})\ \hat{n}_3$	

If B_{33} is made 0, $\dot{x}_3 = 0$ and thus x_3 remains constant at its initial value. Furthermore, if detumbling is initiated with $x_1 = 0$ and $x_2 = 0$, then x_1 and x_2 remain zero throughout the detumbling and the OMV is always in a state of constant spin about its principal axis represented by $\hat{e}_3$ in Fig. 1.

Implementing the control law given by Eq. (23) requires the scalar constant K and the remaining four diagonal elements of B to be specified. To keep controls small, large displacements of the target's mass center from the OMV principal axis aligned with $\hat{e}_3$ must be avoided. This is achieved by specifying the matrix D in the system of Eq. (18) in conjunction with K and B_{44} to coordinate the joint translation with the decay of x_6.

Results

The mass properties of the OMV/target system used in Refs. 3, 13, and 14 are given in Table 1 and will be used here to facilitate comparisons.

Table 2 specifies the initial conditions of the OMV/target system considered in Refs. 13 and 14 and employed here.

Setting the initial joint velocity and acceleration to zero, the remaining system constants were selected as

$$K = 0.001225,\ 1/\text{s}^2$$

$$B = \begin{Bmatrix} +0.046 & 0 & 0 & 0 & 0 \\ 0 & +0.046 & 0 & 0 & 0 \\ 0 & 0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & +0.07 & 0 \\ 0 & 0 & 0 & 0 & +0.02 \end{Bmatrix},\ 1/\text{s}$$

$$D = \begin{Bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.000064 & -0.0048 & -0.12 \end{Bmatrix}$$

B_{44} and K critically damped the x_4, x_6 system with two equal eigenvalues of -0.035. The matrix D resulted by specifying three equal eigenvalues (-0.04) to coordinate joint motion with the cone angle decay. All of these constants were selected to achieve a nearly spin-stabilized state after 300 s of feedback control, which is the maneuver time previously used in Refs. 13 and 14.

The system states during the detumbling maneuver are shown in Figs. 2-4. Figure 2 illustrates the joint motion. The length y_1 [Eq. (17)] is reduced to 0.05% of its initial value at the end of the maneuver, placing the joint very close to the center of the OMV as desired. Figures 3 and 4 show the history of the rates $\dot{\gamma}_1$ and $\dot{\gamma}_2$ ($-x_4$ and x_5) and the angle γ_1 ($-x_6$). The target's spin rate $\dot{\gamma}_2$ is smoothly damped to 0.5% of its initial value during the maneuver. The target's "coning angle" γ_1 is efficiently removed, with the rate $\dot{\gamma}_1$ remaining small throughout the maneuver.

At each integration step, the control vector $\boldsymbol{u}$ was computed from Eq. (23) and the constraint torque and constraint force

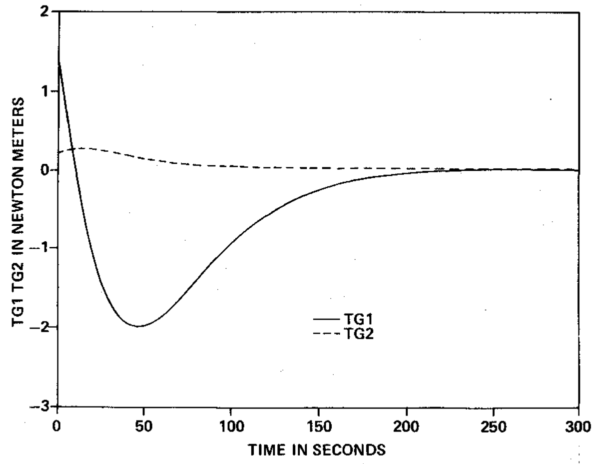

Fig. 6 Internal torque histories.

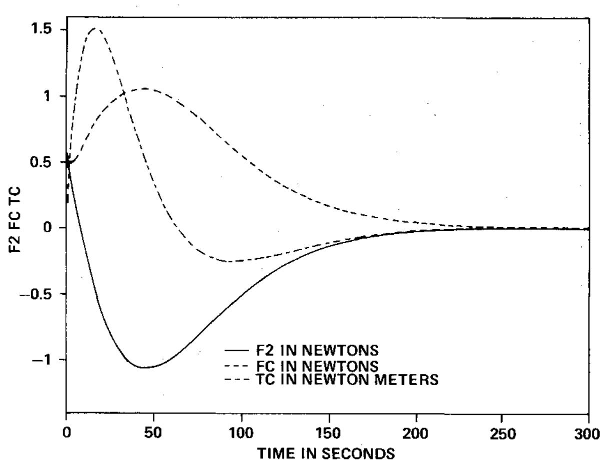

Fig. 7 Constraint load histories.

from Eqs. (9) and (10) with inputs from the system of Eq. (24). The five control histories are shown in Figs. 5 and 6 where

$$[u_1\ u_2\ u_3\ u_4\ u_5] = [T1\ T2\ T3\ TG1\ TG2]$$

$T1$ and $TG1$ are seen to have similar profiles. $T3$ and $TG2$ oppose each other in sign, but otherwise differ primarily only in the first 50 s. Figure 7 shows the constraint loads at the joint during detumbling. FC is the magnitude of the joint constraint force on the OMV and TC the dot product of the joint constraint torque on the OMV with the constraint axis, which forms a dextral set with $\hat{g}_1$ and $\hat{g}_2$. $F2$ is the measure number of the $\hat{e}_2$ component of the joint constraint force and is the magnitude of the force required to translate the joint during detumbling. While the constraint loads may not reach excessive peak values, the peaks are a direct function of the specified control system gains and the constants of the joint motion. Consequently, these peaks will vary with changes in the system parameters.

Conclusions

Using Liapunov stability theory, a nonlinear feedback control law has been developed by which an axisymmetric target satellite tumbling in a state of free spin and precession could be detumbled by an orbital maneuvering vehicle. Global asymptotic stability of the spin-stabilized equilibrium is assured by the Liapunov theory. Employing the control law in the system equations of motion transforms the original nonlinear system of equations into a linear system. System response to gains specified in the control law is easily studied through linear system theory, but the effects of gain variations on control profiles are not as readily determined because the control law is a nonlinear function of the system states along a trajectory through the state space. The control gains used here, for the given initial state, resulted in the orbital maneuvering vehicle maintaining a constant state of pure spin throughout the detumbling process, which may be desirable. However, control gains were selected only on the basis of desired system response. No iterative tradeoff between system response and control profiles was used to modify control system gains.

The results presented here clearly illustrate the complex nature of the detumbling problem. At the same time, the results seem to suggest that detumbling does not require excessive control magnitudes and does not induce excessive structural loads at the point of connection between the target satellite and the orbital maneuvering vehicle. However, these results were obtained without considering the practical constraints on the control system. For example, allowing continuously variable thruster torques in the control law is probably unrealistic. A logical step to advance the work presented here might be to attempt to derive a control law to accomplish detumbling with the same connecting joint system, but with reaction wheels in place of thrusters.

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